DISCUSSION PAPER SERIES

DP14010

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INTERNATIONAL MACROECONOMICS AND FINANCE AND PUBLIC ECONOMICS

ISSN 0265-8003

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Discussion Paper DP14010
Published 18 September 2019
Submitted 17 September 2019

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Abstract

Why can Japan sustain debts above 200 percent of GDP, while Ukraine defaulted on its debt when it was 30 percent of GDP? Answering that question is challenging. First, debt sustainability does not easily translate into operational concepts and indicators. Second, servicing the debt is a strategic decision, the result of a cost-benefit analysis. Thus markets can always, for good or bad reasons, question governments' commitment to face their financial obligations. Third, uncertainty around public debt developments is large and difficult to model. Fourth, not all debts are born equal, as the currency composition, maturity structure, type of creditor and ownership of the debt affect exposure to rollover and liquidity risks. The paper surveys the knowns and unknowns of debt sustainability, including the tools helping us to understand vulnerabilities and to inform our judgment. Instead of embarking on the impossible mission to build a holistic, consistent and broadly-accepted debt-sustainability framework for practitioners, we take the more modest approach to review some of the key economic principles and statistical methods that form today's leading practice in debt sustainability assessments.

JEL Classification: H62, H63

Keywords: debt sustainability analysis, Willingness to pay, default

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Acknowledgements
Debrun: National Bank of Belgium; Ostry and Willems: International Monetary Fund; Wyplosz: Graduate Institute, Geneva. This paper is a chapter in the forthcoming volume Sovereign Debt: A Guide for Economists and Practitioners, Oxford University Press. The authors are grateful to our discussants, Doug Elmendorf and Elena Duggar, and to the volume editors—Ali Abbas, Alex Pienkowski, and Ken Rogoff—for insightful comments. Xiaoxiao Zhang provided superb research assistance. Views expressed are the authors' and should not be attributed to the National Bank of Belgium or the International Monetary Fund.

PUBLIC DEBT SUSTAINABILITY

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Abstract. Why can Japan sustain debts above 200 percent of GDP, while Ukraine defaulted on its debt when it was 30 percent of GDP? Answering that question is challenging. First, debt sustainability does not easily translate into operational concepts and indicators. Second, servicing the debt is a strategic decision, the result of a cost-benefit analysis. Thus markets can always, for good or bad reasons, question governments' commitment to face their financial obligations. Third, uncertainty around public debt developments is large and difficult to model. Fourth, not all debts are born equal, as the currency composition, maturity structure, type of creditor and ownership of the debt affect exposure to rollover and liquidity risks. The paper surveys the knowns and unknowns of debt sustainability, including the tools helping us understand vulnerabilities and inform our judgment. Instead of embarking on the impossible mission to build a holistic, consistent and broadly-accepted debt-sustainability framework for practitioners, we take the more modest approach to review some of the key economic principles and statistical methods that form today's leading practice in debt sustainability assessments.

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[1] Debrun: National Bank of Belgium; Ostry and Willems: International Monetary Fund; Wyplosz: Graduate Institute, Geneva. This paper is a chapter in the forthcoming volume *Sovereign Debt: A Guide for Economists and Practitioners*, Oxford University Press. The authors are grateful to our discussants, Doug Elmendorf and Elena Duggar, and to the volume editors—Ali Abbas, Alex Pienkowski, and Ken Rogoff—for insightful comments. Xiaoxiao Zhang provided superb research assistance. Views expressed are the authors' and should not be attributed to the National Bank of Belgium or the International Monetary Fund.

1. Introduction

Why does Japan defy gravity with gross public debt levels above 200 percent of GDP and others default on a considerably smaller stock of obligations (e.g. 30 percent of GDP in Ukraine in 1998)? This is an example of the vexing question of debt sustainability that this chapter seeks to answer. Doing so requires us to tackle at least four different difficulties:

First, is the definitional challenge. Theory generally equates public debt sustainability with government solvency (i.e. the ability for the public sector to honor all its future financial obligations). However, theoretical clarity does not always translate into operational convenience, in part because sustainability is an inherently forward-looking concept, and, thus, an informed judgment on a known unknown. Thus, practitioners have been struggling to give a concrete meaning to the very notion of sustainability.

Second, standard macroeconomic analysis operates under the presumption that the government is solvent. It seems clear, however, that the benefits of default may in some cases exceed the costs, at least ex-ante, putting into question the credibility of commitments to always repay obligations in full. The very risk of default brings market beliefs into the picture, and with them, the issue of self-fulfilling prophecies whereby mere liquidity crises triggered by senseless panic can lead otherwise solvent governments to default.

Third, there is the operational challenge of modelling uncertainty. The evolution of public debt reflects a broad array of shocks hitting the public sector balance sheet. These range from unexpected policy changes to economic and financial disturbances that can depress government revenues, raise financing costs, or lead to the realization of contingent liabilities. From an operational perspective, analysts must balance the importance of forming a comprehensive view of the relevant risks to debt sustainability with the need to preserve technical tractability and transparency in their assessment. This explains why extensive stress tests and probabilistic models feature prominently in modern toolkits for debt sustainability analysis.

Fourth, not all debts are born equal. Some are more prone to rollover and liquidity risks than others. The currency composition (local vs. foreign), maturity structure (long vs. short term), and ownership of the debt (resident vs. non-resident) matter a great deal because they directly affect exposure to adverse shocks. The type of creditor (private investors, banks, official institutions, …) and debt contract (traded bonds, bank loans, official loans at a subsidized interest rate, …) must also be taken into account when assessing sustainability.[2]

The aim of this chapter is to survey the knowns and unknowns of debt sustainability, including the range of tools at our disposal to understand vulnerabilities and inform what will always remain a difficult judgment call under considerable uncertainty. The chapter builds around the nexus between fiscal policy behavior and the determinants of gross public debt dynamics (mainly interest rates and growth), showing that debt sustainability is as much a political issue as an economic one. The implied complexity has prevented the emergence of a holistic, consistent and broadly-accepted framework for practitioners, and we do not embark on the

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[2] A case in point is the specific debt sustainability framework applied to low-income countries, as these tend to rely mostly on official financing at concessional terms.

impossible mission to build such a framework. Instead, we take the more modest approach to review some of the key economic principles and statistical methods that form today's leading practice in debt sustainability assessments.

The chapter is structured as follows. Section 2 defines debt sustainability, reviewing the basic concepts such as solvency and the deterministic arithmetic of the government's budget constraint. In Section 3, we discuss quantitative assessments of government credibility, exploring the reasons why a government may find itself to be either unable or unwilling to meet its obligations. The notion of debt limit receives particular attention. Section 4 introduces the common tools to capture uncertainty. It looks into the main sources of uncertainty surrounding debt dynamics and shows how they can be incorporated in sustainability assessments. As solvency concerns (founded or not) usually erupt in the form of sudden interruptions in short-term financing, Section 5 discusses ways to include liquidity considerations in sustainability assessments. Section 6 explores several issues that may gain greater prominence in the future, including the role of specific monetary regimes (currency union, reserve currency issuer), the persistence of low interest rates, and the growing interest in broader views of sustainability reflecting the entire public sector balance sheet. Section 7 concludes.

2. Defining Sustainability

Debt sustainability perfectly illustrates the difficulty of deriving simple operational definitions from well-defined economic concepts. A broad consensus exists to consider public debt as sustainable when the government has a high probability of being solvent—i.e. *able to honor its current and future financial obligations*—without having to resort to unfeasible or undesirable policies (see e.g. IMF 2013). However, because solvency boils down to a mere prediction about *future* budget balances over an indefinite horizon, it has no clear operational implication. Thus, the concrete approaches to assess debt sustainability have focused on sufficient (but by no means necessary) conditions for solvency; and since one can think of many such conditions, the debt sustainability literature has inevitably been quite eclectic.

After a brief discussion of the government budget constraint, we use the simple arithmetic of the debt-to-GDP ratio to derive a formal definition of solvency and a common operational condition satisfying the solvency constraint, i.e. the stabilization of the debt-to-GDP ratio. The section concludes with a discussion of a widely used econometric test of debt sustainability proposed by Bohn (1998).

2.1. The Government Budget Constraint

The idea behind any budget constraint is simply that nobody can have their cake and eat it, although this does not have to be the case every period. In modern economies, financial intermediation—mainly through markets and banks—allows some to spend more than their income, but only if others, in the domestic economy or elsewhere, spend less than theirs. The level of interest rates is expected to balance the demand and supply of funds.

For such a system to work, any debt contracted by an agent in deficit must be considered as an asset (wealth) by the agent in surplus. That is why debt contracts must ultimately be honored. In short, solvency is essential to the stability of the system. [3]

The government is a special borrower on several counts. First, it is usually not expected to die or disappear so that there is no obvious end-period when all debts should be repaid. Second, default by the government is a particularly scary prospect because the size of the entity typically entails a considerable destruction of wealth, a collapse in national income, and guaranteed misery for those who cannot insure against such risks, usually the less affluent in society (Borensztein and Panizza, 2009). Third, a government is sovereign. Concretely, this means that (i) it cannot be liquidated (there is no well-defined bankruptcy procedure giving lenders any claim on its assets), (ii) that it can often create fiat money to meet its obligations denominated in domestic currency, and (iii) that it can also raise revenues at discretion by hiking taxes—at least up to the point when tax rates become so toxic for the economy that revenues ultimately fall in response to higher rates (i.e. the Laffer curve effect).

Government's specialness implies that its budget constraint does not bind ex-ante and that servicing the debt is essentially a strategic choice, the outcome of a cost-benefit analysis. This brings political considerations, blurring the conceptually neat distinction between the willingness to service the debt and the ability to do so. For anyone trying to predict whether the government will meet its financial obligations over the foreseeable future, this constitutes a serious complication.

Of course, specialness has its limits. The budget constraint may not bind ex-ante, but it always binds ex-post. Thus, debt sustainability is not about whether the government budget constraint will be fulfilled (it always will) but whether the strategies used to stick to it are feasible and desirable. At the most fundamental level, the solvency requirement rules out default (complete or partial, negotiated or not) as a desirable option. Raising inflation to reduce the real value of nominal obligations (denominated in local currency) is also usually excluded from the set of acceptable strategies to stick to the budget constraint. The "inflation tax" is not only a shadow form of default, it is also hard to envisage in a world that has come to value independent central banks for their success in anchoring inflation expectations to harmless levels. The perceived costs of abandoning monetary credibility often seem exceedingly high and may explain the evidence that countries sometimes prefer an outright default (or to seek debt restructuring) on domestic debt to inflating it away (Reinhart and Rogoff, 2009).

Handling these various considerations, and the interactions between them, requires extensive analysis and, in the end, a lot of judgment. While it is illusory to think that debt sustainability could ever be inferred mechanically from the government's balance sheet, comprehensive frameworks such as those developed by the IMF seek to organize a rich set of relevant data informing that judgment.

[3] The expectation that governments will honor their debt in all states of the world is ultimately what makes their bonds safe. This characteristic has economy-wide benefits, if only for the stability of the financial sector, the viability of pension funds, and the conduct of monetary policy.

2.2. Government Solvency and Public Debt Stability

Since government solvency is the consensual *necessary* condition underlying debt sustainability, it is worth asking what makes a government able to honor its financial obligations in full. Some minimal arithmetic is required to fix ideas and understand why solvency cannot yield an operational definition of debt sustainability.

In any given period t, total government spending must be covered by revenues and bond issuance. To keep the notation as simple as possible, we make the conventional assumption that public debt consists of one-period bonds. The stock of inherited debt (D_{t-1}) must be repaid at the end of the period plus interest due (applying a rate r_t). The period-t government budget constraint thus writes as follows:

$$G_t + (1 + r_t)D_{t-1} = T_t + D_t \ , \tag{1}$$

where G_t is the non-interest (or primary) expenditure and T_t represents total tax revenues.[4] At the end of period t, public debt D_t is the stock of past obligations D_{t-1} to which we add the interest bill $r_t D_{t-1}$, and subtract the difference between total revenues and primary expenditure, known as the *primary balance*: $PB_t \equiv T_t - G_t$.

$$D_t = (1 + r_t)D_{t-1} - PB_t \ . \tag{2}$$

Because the economy's taxable income roughly grows with nominal GDP, it is common to scale the nominal amounts in identity (2) in terms of ratios to nominal GDP (denoted by Y_t). The idea is that if government revenues can grow indefinitely, so can expenditure and debt. Assuming that Y_t grows at an annual rate θ_t, we can transform equation (2) as follows (with lower-case letters denoting ratios to nominal GDP):

$$\frac{D_t}{Y_t} = (1 + r_t)\frac{D_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t} - \frac{PB_t}{Y_t},$$

$$d_t = \left(\frac{1 + r_t}{1 + \theta_t}\right) d_{t-1} - pb_t \qquad (3).$$

At time t, the public debt-to-GDP ratio d_t results from the interest burden of past debt, the economy's rate of growth and the present primary balance.

The impact of the interest bill on debt-ratio dynamics depends on nominal growth. Under the conventional assumption that the interest rate exceeds growth ($r_t > \theta_t$),[5] the debt-to-GDP ratio increases automatically because the rise in GDP (higher denominator) cannot counterbalance the additional debt (higher numerator) that would be required to pay the interest bill with

[4] Non-tax revenues (including interest-sensitive ones, and those related to monetary policy operations) are ignored here for convenience.

[5] In macroeconomic theory, this assumption is known as dynamic efficiency. It ensures that budget constraints are well defined by ruling out Ponzi schemes. However, that condition can be violated in practice, as discussed in Section 6.

borrowed funds. In that case, debt could snowball out of control unless part of the interest bill is funded with own revenues. The resulting primary surplus contributes to lower the debt ratio ($pb_t > 0$), although this might not be enough to stabilize or lower the debt ratio (see below). If instead newly borrowed funds in period t exceed the interest bill, a primary deficit ($pb_t < 0$) further adds to debt in that period.

To fully understand the hydraulics of the government budget constraint, we need to acknowledge the possibility to roll over public debt indefinitely. At the same time, it is intuitively clear that there could never be any "terminal" debt stock the government could conveniently dispose of at some hypothetical "end of times." Nobody in the economy would ever accept holding a bond that could not be realized to finance some future spending (e.g. O'Connell and Zeldes, 1988). Technically, the impossibility to be in debt at the "end of times" is known as a "transversality" condition. Under normal conditions for growth and interest rates, this condition implies that for the government to be solvent, its debt d_t cannot exceed the present value of all *future* primary balances. Equivalently, primary deficits must at some point be fully offset by surpluses. Thus, the government solvency condition writes as follows:[6]

$$d_t \leq \frac{pb_{t+1}}{\left(\frac{1+r_{t+1}}{1+\theta_{t+1}}\right)} + \frac{pb_{t+2}}{\left(\frac{1+r_{t+1}}{1+\theta_{t+1}}\right)\left(\frac{1+r_{t+2}}{1+\theta_{t+2}}\right)} + \cdots \quad (6)$$

The concrete challenge of assessing solvency is immediately clear: given d_t, it amounts to predicting future fiscal policy (primary balances) over an infinite horizon. As if that was not hard enough, the simple deterministic arithmetic above ignores that such prediction is subject to considerable uncertainty surrounding (nominal) economic growth, borrowing costs, and the primary balance itself. The bottom line could be that government solvency is a genuine "known unknown," and that assessing it is "mission impossible" (Wyplosz, 2011).

However, regardless of the immense practical challenges, knowing whether (6) holds or not (without resorting to toxic strategies) is vital. One concrete approach derived from the above arithmetic is to look at the determinants of debt dynamics. This leads to intuitive indicators that are easy to interpret and widely used in debt sustainability frameworks.

From the public debt accumulation equation (3), the evolution of debt over time is given by:

$$\Delta d_t \equiv d_t - d_{t-1} = \left(\frac{r_t - \theta_t}{1+\theta_t}\right) d_{t-1} - pb_t. \quad (7)$$

Changes in d_t are driven by the interest-growth differential, whose impact is directly proportional to the initial debt level, and the primary balance. As governments know that Ponzi strategies (i.e. paying interest with new debt) cannot be sustained forever, they are usually assumed to cater for solvency by generating higher primary balances in response to rising debt.[7] Hence, debt dynamics are shaped by two opposing forces: the debt-increasing power of the

[6] Denoting $R_t \equiv (1+r_t)/(1+\theta_t)$, a more compact expression is: $d_t \leq \sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R_{t+k}} pb_{t+k}$.

[7] Section 3 discusses this assumption in greater detail.

"snowball" of the interest rate minus the growth rate (the interest-growth differential); and the debt-reducing effect of the primary balance.

If we parametrize the response of the primary balance to debt by setting $pb_t = \rho d_{t-1}$ (where $\rho > 0$), we can see from equation (7) that if such a response more-than-offsets the automatic debt buildup that would arise if interest payments were covered with borrowed funds, the debt ratio would revert to some historical mean pinned down by the long-run (or "steady-state") values of the interest-growth differential and the primary balance.[8] In other words, the condition $\rho > \frac{r-\theta}{1+\theta}$ ensures dynamically stable public debt trajectories.

Assessing whether the debt-to-GDP ratio belongs to a dynamically stable trajectory is at the core of debt sustainability frameworks, such as those developed at the IMF (see Annex 1). Although operational challenges remain daunting, the object of judgment (i.e. the stability of the debt path over the medium term) is more palatable than guessing the present value of future primary balances over an infinite horizon.[9]

The focus on short-to-medium-term debt dynamics also allows defining indicators that link debt sustainability to convenient measures of policy adjustments potentially required to preserve it. One such measure is the gap between the actual primary balance and the size required to stabilize public debt at a certain level over a given horizon. In its simplest incarnation, the indicator is the difference between the primary balance that would stabilize the debt ratio between t and $t + 1$ and the projected primary balance for year $t + 1$. The debt-stabilizing primary balance pb^o_{t+1} is easily found by solving equation (7) for $\Delta d_{t+1} = 0$, which yields $pb^o_{t+1} = \left(\frac{r_{t+1}-\theta_{t+1}}{1+\theta_{t+1}}\right) d_t$. The debt-stabilizing primary balance is proportional to the inherited debt level with a proportionality factor given by the interest-growth differential. The resulting year-on-year gap is defined as $pb^o_{t+1} - pb_{t+1}$ (Blanchard, 1990).[10]

While a large gap signals significant challenges to keep the debt ratio under control in the short term, closing the gap in one year may not be feasible nor desirable. Moreover, one year is an

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[8] Note that this stability condition applies regardless of the sign of $\frac{r-\theta}{1+\theta}$ (Bartolini and Cottarelli, 1994) and is therefore robust to situations of persistently negative interest-growth differentials (as analyzed in Blanchard, 2019). It nevertheless remains standard to assume that in steady state, the interest rate is greater than GDP growth (i.e. $\frac{r-\theta}{1+\theta} > 0$).

[9] Note that one class of *theoretical* macroeconomic models—known as the Fiscal Theory of the Price Level—suggest that stable public debt dynamics around a well-defined steady-state is a precondition to ensure price stability when central banks use the interest rate as their policy instrument and public obligations are nominal (see Leeper, 1991; Sims, 1994; and Woodford, 1994).

[10] For instance, if public debt is at 60 percent of GDP and the differential between the interest rate and growth is 100 basis points, a primary surplus slightly below 0.6 percent of GDP keeps the debt ratio constant year on year.

(continued…)

exceedingly short horizon to inform us whether debt is on a stable path or not. Therefore, similar metrics have been defined over a longer horizon. For instance, the European Commission's "S1" indicator calculates the constant yearly adjustment in the *structural* primary balance (i.e. adjusted for temporary influences on the budget, including the economic cycle and "one-off" expenditure or revenue items) needed to reach a given debt level at a predetermined date.[11]

While sustainability indicators capture the size of fiscal adjustment that is eventually required for debt to remain on a stable path, they say nothing about the realism of these hypothetical policies. Yet such realism is at the center of conventional definitions of debt sustainability which stipulate that solvency be maintained without enacting unrealistic or undesirable policies. One way to address this issue is to look at the tax-to-GDP ratio needed to stabilize debt at a certain level over a given horizon (given projected expenditure). The difference with the actual tax ratio may give a better sense of the policy *effort* required to stabilize debt (Blanchard, 1990). The required fiscal adjustment can also be compared to historical norms. For instance, Abiad and Ostry (2005) suggest estimating "fiscal reaction functions" to get a sense of realistic primary balances one could expect in a specific context (as determined by history, external anchors, and institutions' quality). Similar work by Mauro et al. (2015) and Debrun and Kinda (2016) indicates that the debt-stabilizing response of fiscal policy varies with the level of interest rates, long-term growth, and inflation. The IMF DSA template reflects this approach by comparing the projected fiscal adjustment for the country under review to the distribution of observed fiscal adjustments in a large panel of countries.

Of course, realism must also apply to the macroeconomic assumptions underlying projected debt trajectories. Similarly to unrealistic policy effort, overoptimistic projections for growth, interest rates, or exchange rates can create the illusion of a sustainable debt position. The case of Greece discussed in Box 1 illustrates the criticality of realistic macro-fiscal projections to make credible debt sustainability assessments.

Box 1. Greece: A Case of Unrealistic Macro-Fiscal Assumptions

Recent experience with Greece underlines the importance of using realistic fiscal and macroeconomic projections. When the first signs of deep fiscal troubles emerged in 2009, it became clear that major fiscal adjustment was necessary to put public finances back on a sustainable path. At the time, Greece's ability to turn dynamics in the primary balance around was however significantly overestimated and the primary balance undershot projections by an average of 3.2 percentage points per year during 2010-2017 (Table 1).

[11] The algebra is obviously more involved than for the year-on-year gap but remains straightforward. Escolano (2010) provides complete derivations.

Table 1: primary fiscal balance in Greece, May 2010 forecast versus realization

	2010	2011	2012	2013	2014	2015	2016	2017
May 2010 forecast	-2.4	-0.9	1	3.1	5.9	6.0	6.0	6.0
Realization	-5.3	-3.0	-1.5	0.4	-0.0	0.7	3.8	3.7
Forecast – realization	*2.9*	*2.1*	*2.5*	*2.7*	*5.9*	*5.3*	*2.2*	*2.3*

Source: IMF (2010) and IMF WEO database.

The fact that fiscal projections were based upon general growth projections that were overly optimistic (Table 2), was a major contributor to the discrepancy in Table 1. Since both forms of over-optimism endured over time, originally envisaged projections for the debt-to-GDP ratio quickly became unrealistic. At the time of Greece's first IMF program request (in May 2010, see IMF (2010)), it was expected that Greek government debt would peak at 149 percent of GDP in 2013, subsequently declining to 120 percent of GDP by 2020. In reality, debt quickly shot up to about 180 percent of GDP before stabilizing. At the time of writing, the latest IMF projections suggest that Greek debt is highly unsustainable and under the baseline scenario, the debt ratio is projected to exceed 300 percent of GDP by 2080 (IMF, 2017a).

Table 2: real GDP growth in Greece, May 2010 forecast versus realization

	2010	2011	2012	2013	2014	2015	2016	2017
May 2010 forecast	-4.0	-2.6	1.1	2.1	2.1	2.7	2.7	2.7
Realization	-5.5	-9.1	-7.3	-3.2	0.7	-0.3	-0.2	1.4
Forecast – realization	*1.5*	*6.5*	*8.4*	*5.3*	*1.4*	*3.0*	*2.9*	*1.3*

Source: IMF (2010) and IMF WEO database.

The fact that growth over-optimism was already present in Greece's macroeconomic framework *prior to the crisis*, might have contributed to its origination. In October 2008, the IMF WEO predicted that average growth over the years 2009-2012 would be 2.8 percent per year (a number in line with consensus at the time). Because of this relatively benign assessment, neither creditors nor the Greek government seemed overly concerned about Greek debt sustainability and credit continued to flow into the country. Growth over the period 2009-2012 however ended up disappointing by an average of 9.4 percentage points per year, implying that the borrowing which took place during the wave of relative optimism had led to a debt level that was now unsustainable.

Beaudry and Willems (2018) investigates the link between growth (over-) optimism and (over-)borrowing more systematically. They show that more optimistic growth projections typically induce countries to accumulate more debt—a response consistent with the idea of consumption smoothing. Such a response is not without risk though, as Beaudry and Willems also find that countries for which growth forecasts have been overly optimistic in the past, are more likely to develop debt crises in the future. If past borrowing decisions are based upon elevated growth expectations that fail to materialize, it is no surprise that servicing the accumulated debt might become problematic.

2.3. Econometric Approaches to Debt Sustainability

Because the past can reveal useful information about the future, economists have proposed formal econometric tests of debt sustainability using time-series data. These tests can tell whether public debt and primary balance behavior have historically been consistent with solvency. Thus, any forward-looking assessment hinges on the assumption that the future will look sufficiently like the past.

Chalk and Hemming (2000) review early government solvency tests based only on historical data. They note that these tests capture sufficient conditions for solvency. That line of research revolves around the statistical property of *stationarity* of the two relevant time series in equation (6), namely public debt and the primary balance.

The unconditional distribution of a stationary time series does not change over time, implying that a stationary variable has no trend in its mean.[12] In a seminal study, Hamilton and Flavin (1986) argue that if the solvency condition holds, stationarity in the primary balance series implies that public debt is also stationary. Trehan and Walsh (1988) show that even if debt and the primary balance are non-stationary (or integrated), solvency is satisfied if both series move together (are "cointegrated"), with higher debt systematically associated with higher primary balances.

In a celebrated article, Bohn (1998) goes one step further, arguing that tests based purely on time-series properties of debt and the primary balance miss the general equilibrium conditions linking fiscal policy to the rest of the economy. Bohn's "model-based-sustainability" suggests estimating the conditional relationship between public debt and the primary balance. This is done with a single-equation model explaining the primary balance by public debt and *temporary* variations in government expenditure ($\tilde{g}_t$) and output ($\tilde{y}_t$):

$$pb_t = \beta_0 + \beta_1 \tilde{g}_t + \beta_2 \tilde{y}_t + \rho d_{t-1} + \varepsilon_t \tag{8}$$

Bohn showed that a positive conditional response of the primary balance to public debt (i.e. $\rho > 0$) is sufficient to fulfill the solvency condition in a general equilibrium model under reasonable assumptions. This test has been widely used in the literature to assess whether fiscal policy was "responsible" in the sense of being broadly consistent with solvency. For instance, using a large panel comprising emerging-market and advanced economies over a 25-year period beginning in the early 1990s, Mendoza and Ostry (2008) show that government policy seems consistent with fiscal solvency in many countries (not just the United States, as investigated by Bohn).

Of course, a critical issue is the long-term perspective underlying that approach: the fiscal policy response to debt must be sufficiently systematic and stable over time to be meaningful. If that response is positive for a decade but subsequently fades away, no clear inference can be drawn in terms of whether fiscal behavior is consistent with debt sustainability. And indeed, the Bohn condition ($\rho > 0$) does not seem to be satisfied always and everywhere (Mauro et al. 2015). Mendoza and Ostry (2008) also document important differences between advanced and emerging-market economies, including a tendency of the latter to respond more strongly to debt developments than advanced economies, at least up to a certain debt level—around 50 percent of GDP—beyond which the response weakens dramatically. The contrasted experiences of Germany and Japan discussed in Box 2 further illustrate how a stable and positive response of the primary balance to debt shape debt trajectories in otherwise fairly similar economies.

Box 2. A Tale of Two Advanced Economies: Germany and Japan

In many ways, Germany and Japan are similar. They are sizable economies that rely on a strong industrial base favoring export-led growth. Politically, they are stable parliamentary democracies that involve well-established political parties. And, yet, while broadly similar for a long time, the evolution of their public debts could not be more different (Figure 1). While debt ratios had been creeping upward in both countries until

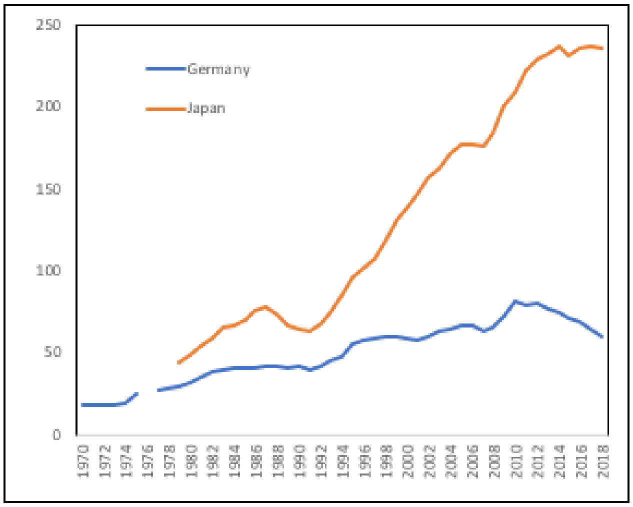
[12] A stationary series has neither a deterministic trend nor a "unit root" (that would imply the absence of convergence to some long-term value).

the late 1980s, the situation then changed radically in Japan. By now, the (gross) debt of the Japanese government is by far the highest among advanced economies. The puzzle is that despite studies consistently showing that there was no fiscal space in Japan, debt kept rising at a breathtaking pace until the mid-2010s without causing the slightest concerns among lenders (the Japanese public itself, for the most part). In contrast, Germany successfully contained the debt buildup, reversing it after 2014.

The debt pickup in both countries around 1990 corresponds to a structural slowdown in growth rates, from an average of 2.5 percent over 1970-1989 to an average of 1.7 percent in 1990-2018 in Germany, and from 4.8 percent to 1.2 percent in Japan. In addition, Germany's reunification also weighed on public finances during the early 1990s. In general, explanations for upward trends in public debt include:

- Implicit or explicit strategy of eventually defaulting.
- Confusion between trend and cycle: the authorities observe lower growth and adopt expansionary policies that fail to deliver the expected sustained boost.
- Conflict with the central bank that responds by raising interest rates.
- Lack of domestic support for fiscal discipline, which leads to destabilizing budgetary cycles when fiscal fatigue sets in.

Figure 1. Germany and Japan: Gross public debt (in percent of GDP)

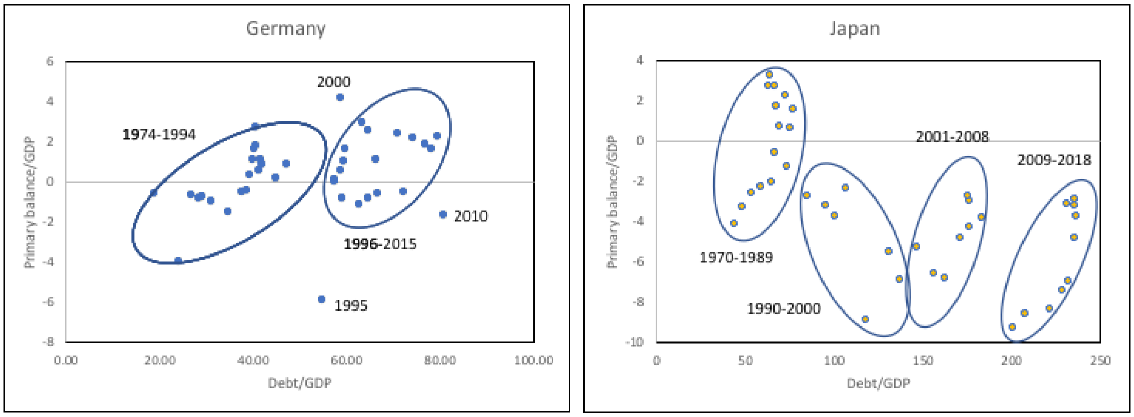


Source: WEO

The first explanation can be ruled out in both cases. This is obvious in the case of Germany but it also applies to Japan whose public debt is mostly held by local financial institutions, the central bank and households. It would just be too costly to default. The second explanation is implausible over the long run but may have played a role for a while in both countries. The third explanation can be justified by central bank statements at various junctures, but there is no evidence that central banks systematically raised their policy rates in response to debt buildup and that higher policy rates are a significant deterrent for deficits.

The fourth explanation would imply a wrongly-signed coefficient on debt in the Bohn's fiscal reaction function (i.e. $\hat{\rho} < 0$). This is not the case in Germany, where the estimated coefficient is positive and highly significant (0.0357). For Japan, however, it is negative and highly significant (-0.300). This might reflect the non-linearity discussed in the text but the scatterplot in Figure 2 suggests an alternative interpretation. While visual inspection confirms that the debt coefficient is negative for the overall period, it points to instability over time, with four distinct subperiods. The first one (1970-1989), displays increasing efforts, eventually successful, at reducing the debt. The following period (1990-2000) is characterized by a clearly negative link between the primary budget balance and debt. Then come two periods of positive relationship (2001-2008 and 2009-2018), whith debt-stabilization efforts of diminishing intensity from one period to the next. The opposite seems to characterize Germany. After 1996, the stabilizing response to higher debt appears to have become more vigorous.

Figure 2. Germany and Japan: Primary balance and debt (in percent of GDP)



Source: WEO

From a practical angle, a major issue is that statistical tests of long-term conditions do not provide guidance on the debt paths and levels that we could safely consider as sustainable. For instance, knowing that the primary balance and the debt level should tend to move together is useful, but it does not rule out rising debt for a long time and to levels most observers, including market participants, would deem "unsustainable." One reason for this is that the Bohn test does not imply any boundary on the primary balance, which makes too many debt paths and levels consistent with solvency. We address this issue in the next section, showing how upper bounds on feasible primary balances defines "debt limits" beyond which the government cannot credibly commit to stabilize the debt.

3. Quantifying Credibility ("Debt Limits")

As made clear above, solvency is secured if the government can *credibly* commit to generate sufficiently large primary surpluses at some point in the future. However, credibility is in the eye of the beholder so that solvency alone does not map into precise properties that any sustainable debt and fiscal policy path should exhibit.

This section first discusses how credibility might be questioned if governments find themselves unable to service their debt. The fundamental reason is that the primary balance cannot rise indefinitely. An upper limit on the primary balance implies that debt also has an upper bound beyond which fiscal policy could not avoid explosive debt dynamics. The section then turns to the strategic dimension of debt sustainability, suggesting that even if it were able to keep debt under control, a government might be unwilling to do so.

3.1. Fiscal Fatigue and Debt Limits

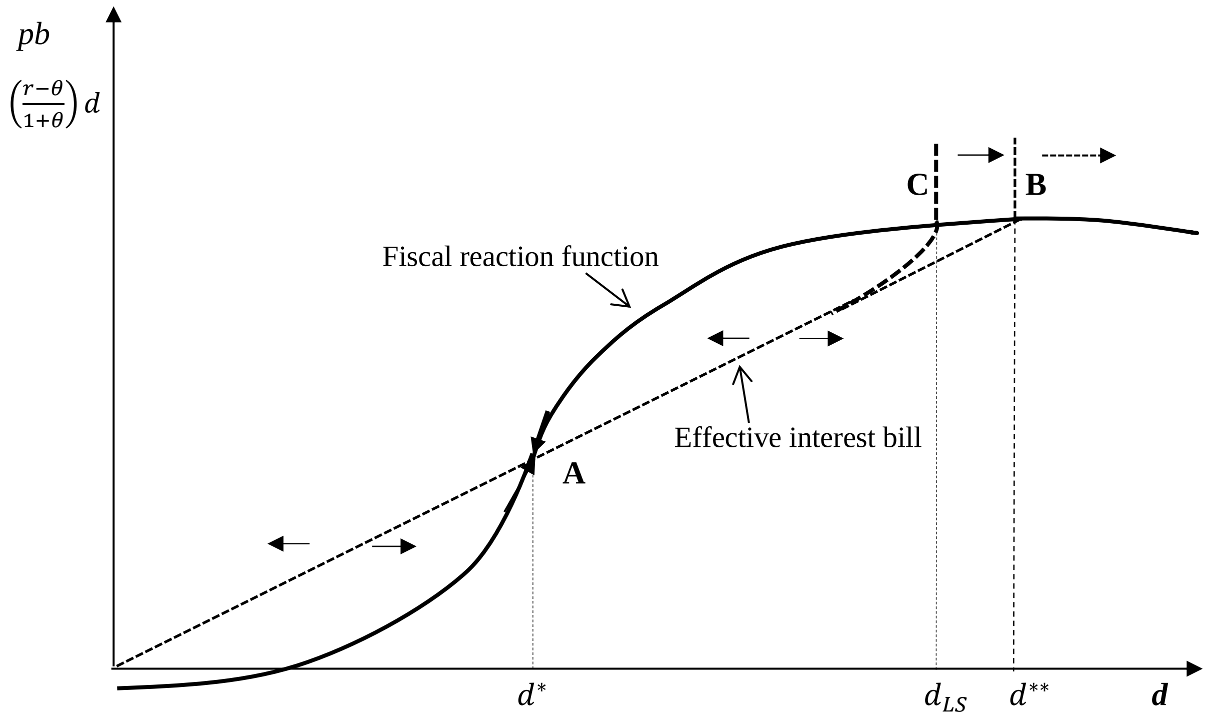
A straightforward way to characterize a debt limit can be found in models that feature "fiscal fatigue" (Ostry et al., 2010; Ghosh et al, 2013). Such models capture the notion that there exists a threshold for the primary balance beyond which the government can no longer keep up with higher interest payments—either because of economic forces (the Laffer curve) or political feasibility (incompressible spending). In such an environment, there will necessarily be a debt level above which debt dynamics become explosive. At that point, the government must either undertake extraordinary fiscal adjustment (where extraordinary means a break with its historical fiscal reaction function) or default on its debt. Default in this setup occurs because of an inability to pay, not for strategic reasons.

Of course, creditors will not be willing to lend to the sovereign at or near the point where default is imminent and will instead demand an increasing risk premium as debt approaches its limit. The general stochastic case, discussed and solved in the aforementioned papers, is rather complicated owing to the joint endogeneity of the risk premium and the default probability, but Figure 3 provides a heuristic treatment. The solid line is a stylized representation of the behavior of the primary balance as a function of debt. At very low levels of debt, there is little response of the primary balance to debt. As debt increases, the balance responds more vigorously, but eventually the adjustment effort peters out as it becomes increasingly difficult and costly to raise taxes or cut primary expenditures. The dotted line in the figure represents the effective interest rate schedule. At low debt levels, the interest rate is the risk-free rate and, assuming that output growth is independent of the level of public debt or the interest rate, this schedule is simply a straight line with slope given by growth-adjusted risk-free real interest rate.

The lower intersection A between the primary balance and interest rate schedule defines the long-run public debt ratio d^* to which the economy is expected to converge. This equilibrium is conditionally stable: if a shock raises debt above this point (but not beyond the upper intersection), the primary balance in subsequent periods will more than offset the higher interest payments, returning the debt ratio to its long-run average.

There is another (upper) intersection as well, however. Abstracting from stochastic shocks and the endogeneity of the interest rate, this intersection B yields a debt limit d^{**} above which debt is unsustainable: if debt were to exceed this point, it would rise forever because, in the absence of extraordinary adjustment, the primary surplus would never be enough to offset the growing debt service. At such a point, the interest rate becomes infinite as the government loses market access and is unable to rollover its debt. In the presence of stochastic shocks to the primary balance and an endogenous response of the interest rate to rising risk, the interest rate schedule of course is not simply the extrapolation of the risk-free rate, but rather bends upward as debt approaches its limit. In such a case, the debt limit d_{LS} is defined by the point C at which there is no finite interest rate that solves the “fixed-point” problem between the default probability and the interest rate (as debt rises, default risk rises which requires a higher yield to compensate investors; and the higher yield in turn raises the default probability).

Figure 3: Determination of Debt Limit



Source: Ostry et al. (2010)

In an attempt to operationalize these theoretical concepts, Ostry et al. (2010) and Ghosh et al. (2013) estimate non-linear fiscal reaction functions on a cross-country dataset covering 23 advanced economies over the period 1970-2007. These papers find evidence to support the notion that fiscal reaction functions display fiscal fatigue, giving rise to debt limits: the relationship between the primary balance and public debt seems to be well-approximated by a cubic function. At low levels of debt there is no, or even a slightly negative, relationship. As debt increases, the primary balance also increases but the responsiveness eventually weakens and then actually decreases at very high levels of debt. This relationship is robust to the addition of a multiplicity of conditioning variables and a variety of estimation techniques. They then use their empirical results to compute *fiscal space*, defined as the difference between current debt ratios and estimated debt limits (Figure 4).[13]

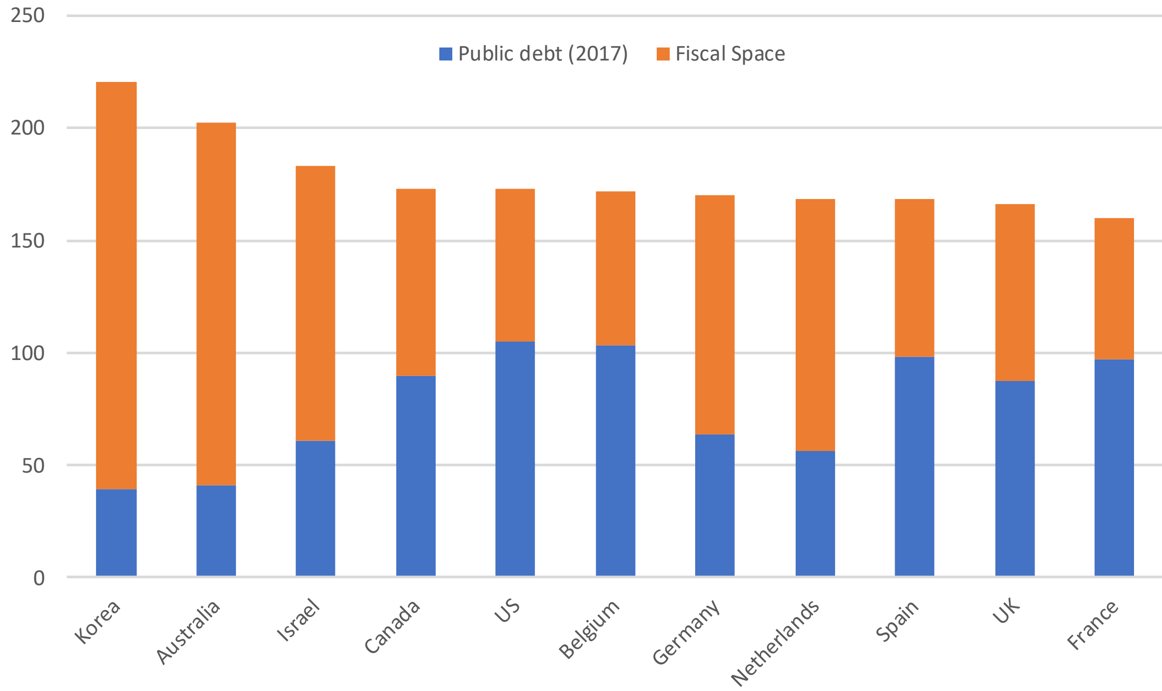
That said, one should keep in mind that debt limits in no way represent normatively-desirable levels of public debt. The potential for surprises argues for normatively-desirable debt levels that are far below estimated debt limits (see Debrun et al, 2019). More generally, a key rationale for low public debt is risk ma*nagement*: the desire for additional margins to cope with unanticipated or contingent risks. As emphasized by Barro (2006), for example, the option value of lower debt is particularly high if there are risks of catastrophic events such that the government would need to ramp up borrowing massively. If debt is high when such a shock

[13] Note that, as argued in Kim and Ostry (2018), the exact nature of the debt contract can affect fiscal space. For a given level of debt, countries will tend to have more fiscal space if their debt stock is of longer maturity. GDP-linked bonds could fulfill a similar role.

(continued…)

occurs, a heavy penalty may be exacted as sovereign premiums rise and, in extreme cases, a shutout from markets would ensue.[14] In other words, lower debt is needed today as insurance against the potential risk of a sovereign crisis tomorrow.

Figure 4. Selected Advanced Economies: Debt Limits and Fiscal Space (in percent of GDP)



Note: The actual gross public debt at end-2017 and fiscal space sum up to the debt limit.
Sources: Ostry et al. (2010), Table 3; and October 2018 IMF Fiscal Monitor.

3.2. Why Do Countries with Sustainable Debts Default?

Governments sometimes default because they cannot service the debt. At other times, they conclude that it pays to default. In a seminal contribution, Eaton and Gersovitz (1981) dissect the strategic choice to pay or not to pay the debt. In their view, defaults result from a cost-benefit analysis. Since defaulting amounts to a capital gain, there must be associated costs. Without them, default would always be the government's preferred option, and it could never borrow. Hence, the very existence of public bonds is predicated on the presence of costs of defaulting. We now discuss what these costs may be.

Market exclusion. Eaton and Gersovitz (1981) assume that the penalty imposed in the event of a default is the inability to borrow. In principle, market exclusion must be permanent, otherwise new lenders would replace the defaulted-upon lenders. Also, the cost of permanent exclusion must exceed the gain from defaulting. As that gain is proportional to the total debt, lenders impose limits on total lending. In practice, the evidence is that exclusion is never

[14] Of course, as emphasized by Reinhart, Reinhart and Rogoff (2015), countries have also availed themselves of a range of heterodox policies to deal with unpleasant shocks in an environment of initially high public debt. But the point about the benefit from relatively low debt, including for the future path of output following a financial crisis (see e.g., Romer and Romer, 2018), remains.

permanent (Panizza et al., 2009), and rarely exceeds a few years (see e.g. Sandleris, 2016; Sandleris et al., 2011; Richmond and Dias, 2009).

In order to explain temporary exclusions, Kovrijnykh and Szentes (2007) suggest that it may be profitable for creditors to resume lending after a default caused by a series of adverse shocks. Eventually, positive shocks will allow the lender to recover some of the defaulted debt *and* to make profits on new loans. To prevent the defaulting government to play lenders against each other, all lenders should be bound by common rules, such as bondholders' committees and *pari passu* agreements enforceable in the lenders' jurisdictions.

Market discipline. Reputation loss is another cost of defaulting because it affects the risk premium demanded by creditors. The empirical evidence suggests that, following a default, borrowing costs rise once the country re-accesses markets. The increase may be steep initially, but it often dies out quickly (Borensztein and Panizza, 2009). This result confirms the conclusion drawn by Bulow and Rogoff (1989) that reputation effects alone are unlikely to sustain lending.

Legal sanctions.[15] Lenders may seek legal authorizations to impose a variety of sanctions. In theory, the defaulter's assets may be seized and trade forbidden, either directly by withholding trade credit or through the banking system to settle payments. In practice, however, courts' ability to constrain sovereigns remains limited. Following the adoption in 1976 by the US of the Foreign Sovereign Immunities Act (FSIA), several countries have followed suite. While the FSIA allows private lenders to sue sovereign entities (governments and their agencies), restrictive conditions apply, which makes the legal firepower far more limited than in the case of disputes among private entities.

A standard court decision is to allow defaulted-upon lenders to seize assets such as pledged collateral, state-owned subsidiaries, exports of state-owned firms, payments for exports, government assets and central bank reserves. While courts have become more open to order asset seizure, defaulting (or would-be defaulting) countries have managed to shield much of their assets.[16]

Collective negotiation. The more effective is a post-default negotiation process, the more lenders can impose costs. In the presence of many lenders, the process may be cumbersome, usually to their detriment. This explains the spread of collective action clauses, which are now standard in many jurisdictions. Recently, however, the emergence of distressed debt funds or vulture funds have complicated matters. These funds first buy securities at deep discount on the secondary market and then initiate litigation to obtain better terms than the investors who reached an agreement with the sovereign through collective action. The holdouts buttress their position by asking to attach assets that are part of the agreement, which inevitably undermines the agreement.

[15] For an extensive review, see Panizza et al. (2009).

[16] A striking example is foreign exchange reserves that are deposited with the Bank for International Settlements, where they are protected by an international treaty.

The precedents have been in flux. While courts have recognized the rights of holdouts to litigate, they have often seen value in upholding agreements between sovereign and a strong majority of bondholders. The situation changed in 2014 when a New York Court backed a group of vulture funds that was blocking the agreement reached after more than a decade between Argentina and a majority of its creditors. One of these funds achieved a return of about 1,000 percent. Faced with this precedent, some countries contemplate legislation against holdouts.

Domestic costs. Defaulting may also entail large domestic costs. Some are related to sanctions, for example on trade or on loss of market access for the private sector, that weakens domestic financial institutions and therefore depresses domestic borrowing (Mendoza and Yue, 2012). A default can also be a bad signal on the government (Sandleris, 2016). It can affect investment decisions by firms, saving decisions by households, lending strategies of banks or trade union militancy. The evidence shows that in all cases, a default can generate domestic disturbances and capital outflows that result in a deep recession and lead to economic and political turmoil.

4. Modelling Uncertainty

As any forward-looking exercise giving a key role to forecasts, debt sustainability assessments require awareness of the uncertainty surrounding medium-term debt projections. Clearly, two otherwise similar countries will be assessed differently if one exhibits highly volatile growth, interest rates, or budget numbers, and the other stable patterns in these variables. Unexpected developments can affect both policy implementation and economic and financial conditions, possibly pushing debt trajectories way off the "baseline." Like the determinants of fiscal behavior, growth and interest rates, the sources of policy uncertainty and macroeconomic volatility are country-specific. DSA tools typically rely on two distinct methodologies to assess uncertainty: stress tests and probabilistic approaches displayed in the form of "fan charts." This section reviews these two approaches.

4.1. Stress Tests

In a purely deterministic world, the determinants of debt dynamics would suffice to inform debt sustainability assessments. In an uncertain world, however, that is not the case. Relative to the baseline, growth could disappoint, interest rates could skyrocket, the exchange rate could collapse, fiscal measures may not be implemented as planned (or have unexpected effects), and contingent liabilities could materialize. While good surprises could also happen, the real concern is that the trajectory of the debt-to-GDP ratio could be significantly more worrisome than envisaged in the baseline.

One approach to account for such uncertainties is to design adverse scenarios that capture particularly bad events for debt dynamics. These so-called "stress tests" aim at gauging the sensitivity of the relevant debt (service) indicators to unfavorable conditions. Typical scenarios include shocks to the primary balance, real GDP growth, the rate of interest, and/or the exchange rate (see e.g. IMF, 2013). The bare-bone stress tests typically consider worse-than-expected realizations for one single determinant of debt dynamics taken in isolation, leaving projections for all other variables unchanged (which is a major drawback of this approach, since it neglects equilibrium effects). As Figure 5 suggests in the case of Italy, a broad range of adverse shocks could derail medium-term debt dynamics, pointing to significant vulnerabilities despite a relatively stable baseline projection. Corresponding simulations for the country's gross financing needs also emphasize potential short-term stress in coping with higher deficits given the large stock of existing obligations coming due (see section 5).

Figure 5. Italy: Stress tests

Note: Primary balance shock: baseline minus half of the 10-year historical standard deviation. Real GDP growth shock: real GDP growth is reduced by 1 standard deviation for two consecutive years, starting in 2019. Interest rate shock: nominal interest rate increases by the difference between the maximum real interest rate over history (last 10 years) and the average real interest rate level over the projection period. Exchange rate shock: the maximum historical movement of exchange rate over ten years. Combined shock: incorporates the largest effect of individual shocks on all relevant variables. Contingent liability shock: one-time increase in non-interest expenditure that is standardized to about 10 percent of banking sector assets. This is assumed to be accompanied by lower growth for two consecutive years by –1½ percentage points, and lower inflation by ½ percent. The primary balance is assumed to worsen by 11 percent of GDP in 2019.
Source: IMF (2019).

Simple stress tests are usually calibrated on country-specific circumstances as described by unconditional distributions of relevant variables. Calibration should typically strike a plausible balance between the intensity of the shock and its persistence. For instance, real growth could be assumed to be one-standard deviation below the baseline for 2 consecutive years or two-standard-deviations below the baseline for 1 year only, depending on the expected anatomy of a growth shock in the specific circumstances facing the country.

Beyond macroeconomic variables, stress tests are also used to analyze the impact of the materialization of contingent liabilities. These are potential liabilities, such as the need to cover losses in state-owned enterprises (SOEs), systemically important private companies, or public-private partnerships (PPPs).[17] Proper calibration requires extensive information about the government's on- and off-balance sheet operations, including exposure to potential SOEs losses, the size of the financial sector (which is of systemic importance in most countries), and the stock of PPPs. While countries following leading international practice in fiscal transparency publish detailed reports on contingent liabilities, this is not the case everywhere, and most stress tests end up being grossly calibrated (e.g. a one-off shock of x percent of GDP). While well-designed stress tests should in principle inform analysts about sensible boundaries to potentially bad realizations of debt trajectories, they remain a deterministic exploration. In other words, any individual stress scenario, regardless of the care and sophistication underlying its design, has zero chance of ever materializing exactly. Shortcuts consisting of standardized stress tests do save time and resources, but they may come at the cost of being mostly irrelevant. Indeed, knowing the impact on the projected debt path of a one-standard-deviation reduction in GDP growth for one year (while nothing else happens in the economy compared to the baseline) is not particularly informative.

4.2. Fan Charts

A more comprehensive approach to assess uncertainty is to prepare a very large number of different scenarios to obtain distributions of possible debt outcomes for each year of the forecast. Such information can be summarized in the form of a chart showing these distributions around the baseline (median) debt path. Those so-called "fan charts" not only give a more informative visual of the uncertainty around debt forecasts, but they also allow for an explicitly probabilistic analysis of debt sustainability (allowing for statements saying that public debt has a less than 10 percent probability of reaching its official target by the time of the next election, which sends a very clear message to voters and market participants).

Fan charts allow showing how an economy's intrinsic volatility—to the extent that is revealed by its own history of shocks—can affect the riskiness of its public debt level. For the sake of illustration, take two advanced economies (Italy and Portugal). For both, we can estimate a simple empirical model providing information on average relationships between the determinants of debt dynamics—namely GDP growth, the interest rate, and the primary budget balance—as well on the typical volatility of these variables given the estimated relationships among them. According to the empirical model, both countries have similar steady state growth-adjusted interest rates, but Portugal faces a higher volatility of growth and interest rates than Italy.

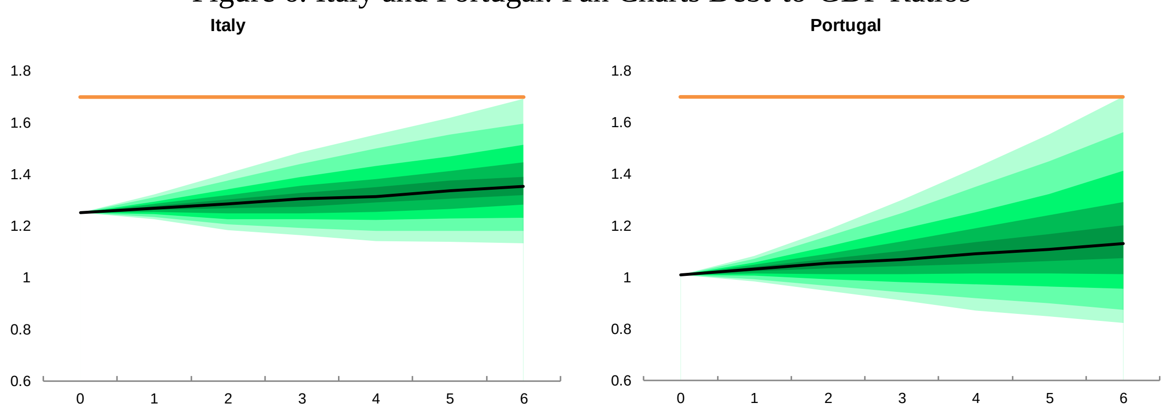
The resulting fan charts are quite different and can be used to illustrate the intrinsic riskiness of each country's public debt (Figure 6). First, observe that the wider the fan, the greater the

[17] A public-private partnership is an arrangement in which a private party provides a public service (being paid for by the government). These arrangements have significant fiscal implications for governments—not only as the PPP contract might require the government to purchase the provider's services for a certain amount of time, but also because the PPP might end up in distress (bringing significant transaction/renegotiation costs).

uncertainty surrounding public debt ratios, with each color band capturing a probability mass of 10 percent, except the two extreme, light-shaded bands, that each represent an area where the debt ratio has a 5 percent chance of materializing. One obvious use of a fan chart is to calculate the probability to reach a certain debt level at one point into the future. For instance, eyeballing the chart for Portugal shows that it is has a roughly 30 percent chance (i.e. two colored bands below the median) of seeing its debt ratio stabilize or fall below its initial level after 6 years.

Another useful application of such charts is to help countries develop a risk-management approach to fiscal policy. In Figure 6, each chart is built with a hypothetical starting debt level (in year 0) such that there is a 5 percent chance of reaching or exceeding the country's debt limit—represented by the horizontal line and taken from Ostry et al. (2010)—after 6 years. The lower volatility of public debt determinants in Italy explains why this starting public debt level is around 120 percent of GDP, some 20 percentage points higher than the equivalent debt level for more volatile Portugal. In other words, the fan chart suggests that while a debt ratio of up to 120 percent of GDP is no cause for concern to a stable economy, it might be much more worrisome for country more exposed to shocks.

Figure 6. Italy and Portugal: Fan Charts Debt-to-GDP Ratios



Note: For each country, the initial debt ratio at year 0 is calculated to correspond to a 5 percent probability of exceeding the median debt limit for the sample of advanced economies considered by Ostry et al. (2010) at the end of a 6-year forecasting horizon. The limit is depicted by the horizontal orange line. Each colored band around the median projection represents a 10 percent probability mass for projected debt trajectories to belong to the corresponding interval. The two extreme light-colored bands capture a 5 percent probability mass. The fan charts show the distribution of 1,000 debt projections and were generated using a variant of the Celasun et al. (2006) routine.
Source: Debrun et al. (2019).

That said, fan charts are only as informative as the inputs and methods used to generate them. There are indeed many ways to simulate thousands of randomly generated debt paths, and they can greatly differ in terms of information contents. Transparency about the "black box" behind random simulations is therefore essential.

In a nutshell, there are two polar approaches to build a probabilistic DSA. A simple but rather crude method is to randomly generate alternative debt paths that reproduce past forecast errors. This "reduced-form" approach thus shows analysts how uncertain their current assessment is if they can be assumed to remain as wrong in the future as they were in the past. At the other end of the spectrum, empirical relationships between all relevant variables for debt dynamics (mainly growth, interest rate, and exchange rates) can be estimated and used to generate a series of shocks over the forecasting horizon as well and forecasts for the determinants of debt consistent with those shocks (see Penalver and Thwaites, 2006; Garcia and Rigobon, 2004; and Celasun et al. 2006).[18]

Although it is more demanding in terms of data requirement and maintenance, the second approach is a priori much richer. First, econometric models such as Vector Autoregressions (VAR) are well suited to (i) capture the dynamic linkages between the determinants of debt as well as their underlying steady state values, (ii) to generate plausible sets of random disturbances, and (iii) to produce consistent forecasts for all relevant variables feeding into the debt accumulation equation. The second key advantage of a model-based approach is the possibility to incorporate an estimated fiscal reaction function, since governments tend not to stay idle in the face of rising public debt. An estimated reaction function also allows to incorporate empirically plausible shocks emanating directly from the budget process and to account for more structural dimensions of a country's capacity to generate primary surpluses, such as good institutions. Table 3 below compares stress testing to the probabilistic DSA.

Table 3. DSA and Risk Assessment

	Deterministic stress-testing	Probabilistic approach (model-based)
Diagnostic based on...	...a few stylized, isolated shocks; exogenous policies.	...many random shocks drawn from an estimated joint distribution; endogenous fiscal policy.
Calibration of shocks	Fraction or multiple of historical standard deviations of underlying variables.	Based on the estimated joint distribution of disturbances.
Output	Large temporary shocks provide a probabilistic upper bound to the debt ratio; small permanent shocks delineate interval of most probable outcomes.	Frequency distributions of the debt ratio over time, "fan charts." Gives a sense of the most likely range within which future values of the relevant debt (service) indicators are likely to lie.
Main advantages	Amenable to standardized stress tests across countries; low data requirement.	Better reflection of country specificity (in terms of shocks and fiscal policy behavior); explicitly probabilistic output.

Sources: Adapted from Celasun et al. (2006).

5. Incorporating Liquidity

So far, this chapter has focused on solvency, which is by essence a medium-to-long-run concept. As such, it largely ignores constraints that may bind in the short-term and that may jeopardize a debtor's ability to honor financial obligations. Liquidity problems, as they are

[18] A host of intermediate options exist to generate random shocks and the corresponding debt paths, including ad-hoc shock distributions.

known, have the same effects as the sudden realization of insolvency: default, restructuring, or other expedients.

As noted by Wyplosz (2011), the IMF (2002) definition of sustainability goes beyond pure solvency issues and covers circumstances typical of illiquidity. First, by considering “major corrections” in fiscal policy as inconsistent with debt sustainability, the definition implicitly captures what happens in a liquidity crisis, when in the absence of new financing at reasonable conditions, public spending should instantly match revenues to stick to the period budget constraint. Second, the explicit reference to the “costs of financing” acknowledges the role of market expectations and risk aversion, as reflected in sovereign risk premiums. By referring to sustainability as the ability to service debt, IMF (2011) effectively lumps together solvency and liquidity.

It remains the case that a perfectly solvent government can suffer from liquidity crises, and an insolvent one can go on for a long time before hitting the wall of illiquidity. This is the ugly face of the so-called multiple equilibria (Calvo, 1988). As long as lenders’ expectations converge on good outcomes (solvency/sustainability), borrowing costs can remain low enough for public debt to stay on a sustainable trajectory. However, if for some reason, views about the riskiness of a country’s public debt change, liquidity stress can suddenly arise, borrowing costs explode, and solvency can instantly become a problem. The self-fulfilling nature of sovereign debt crises complicates sustainability assessments.

Clearly, liquidity must be an important consideration in any comprehensive debt sustainability assessment. Here too, judgment is central, and it concerns lenders’ willingness to cover the government’s gross financing needs (that is the sum of the deficit and rollover needs) without sharp increases in risk premiums. To inform that judgement, indicators of the risks surrounding the debt trajectory will prove useful. In addition, detailed information about the debt structure in terms of maturity, bondholders’ profile (domestic vs. foreign), the repayment schedule (smooth vs. lumpy), and the quality of debt management will help obtain a more reliable forecast of gross financing needs and a better understanding of refinancing risks.

To assess liquidity risks in practice, the literature has relied upon so-called early warning models, as well as analyses of sovereign spreads. We now describe each approach in greater detail.

5.1 Early warning models

The early warning literature is rooted in studies that aim to find the determinants of fiscal stress episodes, typically defined as instances of a default, a restructuring, or an IMF-supported program of significant size. By aiming to explain general episodes of fiscal stress, such studies also capture crises that are predominantly caused by solvency-related considerations. But as most fiscal crises have an important liquidity component as well, contributions to this literature are thought to carry important lessons for liquidity-related aspects of debt sustainability.

An early contribution to the early warning literature was made by Manasse et al. (2003). Combining data from 47 emerging markets over the period 1970-2002 with logistic regression, they find that fiscal crises are more likely in the presence of high external debt, high short-term debt, high debt-service payments, a negative current account balance, tight US monetary policy, low real GDP growth, high inflation (volatility), as well as political uncertainty.

Empirical analyses that explore the determinants of fiscal stress can be used to estimate the probability of stress given a country's characteristics. As the debt level is a determinant of that crisis probability, thresholds above which debt (or debt service) is deemed unsustainable can be inferred. This is the approach underlying the IMF's debt sustainability framework for low-income countries (see IMF, 2017b). The idea is to set a cutoff probability π^* above which the risk of a fiscal crisis occurring (as implied by the regression equation) is deemed too high.[19] One can then back-out the associated threshold values of the debt (service) indicators which would imply a π^* probability of a fiscal crisis, given average values for other crisis determinant in the equation. Table 4 shows the resulting debt thresholds in IMF's current debt sustainability framework for low-income countries.

Table 4. Thresholds in the IMF's debt sustainability framework for low-income countries

Debt carrying capacity	PV of PPG external debt		PPG external debt service		PV of total public debt
Weak	30% of GDP	140% of exports	10% of exports	14% of revenue	35% of GDP
Medium	40% of GDP	180% of exports	15% of exports	18% of revenue	55% of GDP
Strong	55% of GDP	240% of exports	21% of exports	23% of revenue	70% of GDP

Note: Debt carrying capacity is country-specific and determined by a country's score on a composite indicator, combining the quality of institutions, its growth rate, remittances, reserve levels, and world growth. PV = present value. PPG = public and publicly guaranteed.
Source: IMF (2018).

5.2 Sovereign spreads

As a liquidity crisis is characterized by a sovereign's inability to borrow at reasonable rates, liquidity risks can also be gauged from sovereign spreads. These spreads, often closely watched by financial market participants, can be obtained from bond prices or from Credit Default Swaps (or CDS, financial agreements whereby the seller guarantees to compensate the buyer in case of default on an underlying debt contract). Of these two, the latter offers the most precise signal of default because bond spreads may also embed other information such as inflation expectations (Aizenman et al. 2013). However, for countries that borrow in foreign currency (e.g. US dollars or euros, as many emerging markets do), the inflation premium can be ignored. Consequently, many studies analyzing sovereign spreads in emerging markets have used the EMBIG spread index.[20] Thus, both CDS and bond spreads embed a probability of default that can be backed out by making assumptions on 'loss-given-default' and on the (time-varying) degree of risk aversion of creditors.

The most relevant empirical question in the context of this chapter is whether the conventional indicators of fiscal health have the expected influence on sovereign spreads. The literature analyzing the determinants of sovereign spreads goes back to Edwards (1984), who find a positive but statistically weak association with the debt- and debt-service-to-GDP ratios. By

[19] These probabilities can for example be chosen using data on past fiscal crises, with the objective of minimizing erroneous predictions, i.e. missed crises and false alarms.

[20] JP Morgan's emerging markets-focused EMBIG database includes Brady bonds, Eurobonds, traded loans denominated in US dollars, and local market debt instruments. Only issues with a remaining maturity of 2.5 years or more (and face value greater than USD 500 million) are included (see JP Morgan (1999) for details).

contrast, higher investment and international reserves (also scaled to GDP) tend to reduce spread, with the latter playing a particularly big role.

While the variables included in Edwards' regression mostly capture a country's *ability* to service its debt, a country's *willingness* to do so is important as well (Bulow and Rogoff, 1989). Proxying willingness by the level of external payment arrears, Boehmer and Megginson (1990) find that countries that signal a reluctance to service debt (by accumulating arrears) face higher spreads. Using bond-based primary yield data,[21] Min (1998) identifies a wider set of variables to play a role in determining the spread—including the terms-of-trade, the real exchange rate, the rate of inflation, and the level of net foreign assets. Like Edwards (1984), Min (1998) does not find a significant role for fiscal variables in explaining spreads. Other studies report mixed results.

However, once one accounts for *the composition* of fiscal policy, a clearer picture emerges: bond markets seem to distinguish between government spending and government investment—with Peppel-Srebny (2017) reporting that a higher deficit solely due to higher public investment lowers borrowing costs. This suggests that markets believe that the return on public investment improve the sustainability of a given debt level. Akitoby and Stratmann (2008) moreover find that revenue-based adjustment reduces spreads more than spending-based adjustment, while debt-financed spending widens spreads. This should serve as a warning that debt limits and budget balances thresholds should be taken with a grain of salt.

6. Emerging Issues

Despite the many conceptual and practical complexities related to public debt sustainability, it is still assessed using remarkably blunt tools that combine medium-term debt projections and basic indicators of the uncertainty surrounding those projections. In this section, we draw attention on 3 issues that might prove increasingly relevant in the foreseeable future.

6.1 Debt sustainability in members of currency unions

The recent episodes of acute sovereign debt stress in the euro area have been powerful reminders that governments operating in a currency union are special in at least two important aspects. First, monetization is not an obvious way out for them. Even if a new national currency could be created as an expedient, these countries would still have to confront the fact that their debt would be denominated in what would effectively become a foreign currency. Second, the members of a currency union might also be more likely to benefit from explicit or implicit bailouts or external guarantees. Indeed, because of the public good dimension of debt sustainability in any currency union, the will of the union's members to preserve the stability of their shared currency would void the credibility of no-bail-out commitments. In practice,

[21] As argued by Eichengreen and Mody (2000), looking at secondary spreads is preferable, and care should be taken in using primary yields. The reason is that results from primary issuances are likely to suffer from a selection bias: when financing conditions toughen, riskier borrowers might drop out of the market and not place any new bonds. As a result, it is possible that poor market conditions lead to a situation in which primary and secondary spreads move in opposite directions.

many analysists in the business of gauging debt sustainability tend to treat members of currency union differently (see Ghosh et al. (2013) for a formal analysis).

On balance, participation in a currency union is often considered as negative for debt sustainability. For instance, a rating agency like Standard and Poor's assigns lower ratings, all else equal, to debt issued by currency union members, citing obstacles to central bank backstop. Of course, countries might be reluctant to call on last-resort central bank lending in the face of sovereign stress. However, in stressed financial conditions, the very absence of such monetary backstop might increase a government's exposure to self-fulfilling debt crises—i.e. a situation where higher debt causes higher lending costs which ultimately makes it impossible to stabilize the debt. Corsetti and Dedola (2016) show this, arguing that central bank purchases of government debt (in exchange for currency and/or reserves) amount to swapping a claim subject to default for another one with guaranteed face value (central bank money). Since money is subject to an inflation risk, an institution with strong anti-inflationary credentials is in a better position to provide such backstop, which in the end can reduce the risk of self-fulfilling prophecies to the point that no actual debt purchase takes place in equilibrium.

Although the presence of such a monetary backstop seems easier to achieve in countries with their own currency, the ECB commitment to do "whatever it takes" to save the euro is not materially different from a lender-of-last resort function.[22] The announcement of the so-called Outright Money Transaction (OMT) Program in August 2012 seems to have worked exactly as intended. The risk of self-fulfilling debt crises in the euro area has abated even though the program never had to be activated (Saka et al., 2015).

6.2. Low interest rates

As shown earlier, the link between the intertemporal budget constraint and conventional approaches to debt sustainability is based on the premise that the (risk-free) interest rate exceeds the economy's growth rate. Absent this dynamic efficiency condition, the government budget constraint does not really bind. In terms of debt sustainability, it means that the debt-to-GDP ratio can be stabilized or even decline without forcing the government to run a primary surplus. In that sense, Ponzi behaviors can be consistent with debt sustainability.

In reality, episodes of negative interest-growth differentials are the norm more than the exception. For the United Sates Federal Government, Ball et al. (1998) shows that the effective interest rate on public debt—measured as the ratio between the interest bill and the debt stock—was below nominal growth rates on average during 1871-to-1992, 1920-to-1992, and 1946-to-1992. Relatedly, Blanchard (2019) shows that the 1-year US Treasury bill rate has only consistently exceeded the nominal growth rate of the economy during the period extending from the late 1970s until 1990. Finally, for developing economies, Escolano et al. (2017) document large and negative differentials, reflecting mainly negative real interest rates in these countries.

[22] ECB President Mario Draghi also observed in his 2014 Jackson Hole Luncheon Address that "public debt is in aggregate not higher in the euro area than in the US or Japan. [T]he central bank in those countries could act and has acted as a backstop for government funding. This is an important reason why markets spared their fiscal authorities the loss of confidence that constrained many euro area governments' market access."

Financial repression—a mix of regulatory measures creating a captive domestic market for government bonds—is often cited as the main culprit for this situation (Reinhart and Sbrancia, 2015). This was certainly the case in industrial economies during the decades that followed World War II, and it still is by and large the case in many developing economies. However, this explanation is difficult to square with the persistence of negative interest-growth differentials in financially-open, advanced economies during much of the 2000, and certainly since the 2008 financial crisis.

Regardless of whether very low interest rates are here to stay, the implications of this situation for debt sustainability analysis are important and require a discussion beyond the obvious aspects of debt arithmetic. Blanchard (2019) provides a comprehensive analysis of public debt in a low interest environment. He concludes that even though low rates might appear to make public debt a free lunch, there remain welfare costs associated with high debt, albeit smaller than they would be with higher rates. Perhaps the strongest cautionary word against letting public debt grow to very high levels is the risk of self-fulfilling prophecies that invariably come with it. Aside the issue of multiple equilibria, there is a distinct risk that negative interest-growth differentials might quickly reverse as soon as governments are perceived as deliberately engaging in "Ponzi-gambles." (Ball et al., 1998). In the end, the most important item on the researchers' agenda may well be to refine our understanding of public debt limits, including how financial markets and the rest of the economy react to a government approaching such limits. As long as a country has substantial fiscal space left, Ostry et al. (2015) argue that governments should not actively pay down the debt by running overall budgetary surpluses (because the insurance benefit of lower debt in such cases is likely to be smaller than the efficiency losses from temporarily raising taxes or cutting productive spending); instead they should allow growth or non-distortionary revenues (such as privatization receipts or royalties) to organically reduce debt ratios.

6.3. Beyond Debt

Like all economic agents, the government has a balance sheet, an account that collects the *stock* of all assets and liabilities of the public sector. However, unlike most private agents, and certainly listed companies, it is often difficult to know what that balance sheet really looks like, either because it is not published or not even constructed. Ignoring the government balance sheet is more than a mere issue of fiscal transparency. It forces us to ask whether the focus on *gross* public debt—only one component of the balance sheet—might not be too be too narrow.

Assembling a government balance sheet is a daunting task. The asset side includes items like the present value of future tax revenues, financial assets, publicly owned natural resources waiting to be extracted, and non-financial assets like public infrastructure, national parks, architectural wonders, and cultural treasures, all of which have no market value and whose price is consequently unknown. Aside gross public debt, liabilities include pension obligations and other civil service benefits, clearly big-ticket item in countries with unfunded pension systems. The difference between assets and liabilities—the "bottom line"—is the government net worth.

Like solvency, net worth is a theoretically clear-cut notion, and a conceptually attractive basis to define "sustainability." And like solvency, net worth is fully intertemporal and defined over the very long-term. Arrow et al. (2004) suggested that a *non-decreasing net worth* is the right concept of sustainability.

However, like solvency, achieving a non-decreasing intertemporal net worth (INW) has no operational meaning and is arguably too far from anyone's immediate concerns. This likely explains why most countries do not explicitly refer to net worth in the framework guiding the conduct of fiscal policy. For the sake of transparency, more and more countries publish balance sheet and the related analysis. However, the very fact that nobody seems concerned with abysmally negative numbers for net worth is telling. Either they do not realize that they are contemplating sovereign insolvency, or nobody (especially bondholders) cares.

Aside conceptual considerations, the impracticalities related to INW are comparable to those associated with solvency.[23] Data availability and valuation issues are pervasive and the intertemporal nature of INW makes it highly sensitive to assumptions about discount rates, just as the intertemporal budget constraint can be fulfilled with small variations in long-term growth or interest rate assumptions.

In practice, interest in the balance sheet approach suggests considering net public debt instead of gross debt to assess sustainability. For countries with significant liquid financial assets, DSA tools could arguably be run using a net debt metric, an approach supported by the IMF when a country's own fiscal framework uses net debt as a reference (or anchor). That said, question marks remain about whether fire sales of state assets would be feasible to cope with severe liquidity stress, even if the capital losses that such sales could entail should be compared to the costs of outright default.

Regardless of the value of the INW for sustainability, a complete government balance sheet gives a better grasp on the risks facing the public sector. Shocks to the balance sheet are often absorbed by public debt, and the design of stress tests could only benefit from a reliable balance sheet and a careful assessment of the related risks (Clements et al., 2016).

7. Concluding remarks

Assessing public debt sustainability is as critical as it is complicated. It is critical because unsustainable debts often end up in some costly combination of default, high inflation, and a broken financial system. It is complicated because sustainability is inextricably linked to solvency, that is the government's ability to honor all its current and future obligations. Thus, sustainability is a purely forward-looking concept, and assessing it amounts to making a prediction about the unknowable future.

As much as the consequences of insolvency are dramatic and visible, solvency cannot be precisely pinned down in real time with well-defined indicators, such as the debt-to-GDP ratio

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[23] Solvency and the INW are intimately linked. For instance, the European Commission estimates the INW as the difference between the current net worth and the present value of all future primary balances *required to fulfill the intertemporal budget constraint* (the so-called S2 indicator). The Commission's S2 sustainability indicator is simply the wedge in the intertemporal budget constraint (i.e. the difference between current gross debt and the present value of all future primary balances).

or the share of tax revenues allocated to debt service. And even though we can identify critical values of such indicators beyond which a government could be deemed *unable* to pay (e.g. a debt limit), we would still miss the fact that the decision to default (explicitly or implicitly) has a *strategic* dimension informed by a non-trivial cost-benefit analysis and shaped by political constraints. While conceptually neat, the difference between a government's *ability* to pay and its *willingness* to pay is difficult to capture in practice. Better understanding the determinants of (past) debt crises could nevertheless help identify critical debt thresholds that also reflect strategic considerations.

Faced with conceptual fuzziness and multi-layered complexities, practitioners have developed simple sustainability frameworks aimed at informing their judgment. These frameworks typically build on (i) medium-to-long term projections for relevant debt ratios, (ii) indicators of the uncertainty surrounding these projections, and (iii) indicators of potential liquidity stress. Since the turn of the century, the DSA frameworks used at the IMF have evolved to reflect both accumulating experience and progress of applied research. In particular, the treatment of uncertainty has grown more sophisticated to include ex-ante assessments of the realism of the underlying macro-fiscal forecasts as well as probabilistic tools (fan charts) complementing or replacing traditional stress tests.

One safe prediction looking forward is that preparing credible debt sustainability analyses will remain highly challenging. First, the persistence of interest rates below the nominal growth rate of the economy relaxes budget constraints to the point of making Ponzi games consistent with stable or declining debt ratios. This forces economists and practitioners to think hard about what a plausible debt limit could look like in such an environment, a key question being how market expectations could turn around and bring interest rates back above economic growth. Second, the aftermath of the 2008 financial crisis has emphasized the critical role that credible central banks can play in stabilizing sovereign bond markets and mitigating the risk of crisis despite high and rising debt levels. Beyond central banks, the behavior of bond investors warrants due consideration. The existence of a stable demand for assets considered as safe—e.g. because of the strong home bias of a large domestic investors' base or the reserve-currency status of the country—certainly matters when assessing debt sustainability. Third, as DSA frameworks evolve to incorporate more sophisticated techniques (like probabilistic methods), the resulting opacity should not make us lose the intrinsic value of simplicity when communicating about debt sustainability.

Annex 1. Debt sustainability analysis at the IMF

Public debt sustainability analyses play a key role in the work of the IMF: both in cases of surveillance (where the analysis is used to inform policy advice), as well as for IMF lending decisions (since the IMF is in principle banned from lending to a country if it believes public debt to be unsustainable).

To assess the sustainability of debt, the IMF employs two different frameworks: one for countries with market access ("MACs", which focuses on total public debt) and one for low-income countries ("LICs", where DSA focuses upon external public and publicly guaranteed debt). LICs tend to rely more on concessional financing, implying that the nominal value of debt is not necessarily a good indicator of their actual debt burden. Consequently, the IMF's debt sustainability framework for low-income countries places *the present value* of public debt at its core. The present value is inferior to its nominal value when the loan is provided at below-market interest rates or is accompanied by a grace period (during which the debtor is relieved from making repayments, without accumulating interest).

Central to the IMF's debt sustainability frameworks for both types of countries are debt dynamics equations (discussed in Section 2.2). Combined with forecasts for key macroeconomic variables, they yield projections for the debt ratio going forward. An assessment is subsequently obtained by analyzing this projected path and judging whether it passes the "sustainability bar". For both MACs and LICs, that assessment is guided by econometric analyses of past episodes of fiscal stress. Those approaches can either lead to "debt thresholds" (critical ratios, typically in terms of debt-to-GDP or debt service-to-GDP, beyond which debt sustainability is deemed in doubt) or indicators conveying the likelihood of future debt distress (for which one can set a tolerance level). Both tolerance and threshold levels are determined to minimize a weighted average of the rate of false alarms and missed crises over the sample period.

At the same time, both frameworks recognize that liquidity factors play an important role as well (see Section 5). Overall, the IMF's sustainability assessments are not solely informed by projections for debt ratios but by a broad range of indicators that take liquidity considerations into account. Particular attention is paid to indicators like the change in the share of short-term debt, liquid assets available to the government, spread levels, and gross financing needs (defined as the amount of financing required by the government, consisting of the overall deficit, amortization, and funds needed to address possible realizations of contingent liabilities). Gross financing needs and spreads often show significant co-movement, particularly around crises.

Finally, both IMF DSA frameworks also take uncertainty into account—particularly with respect to future paths for debt and debt service indicators. They do this through both fan charts and stress tests. While fan charts seem conceptually superior (see Section 4), they also require more data inputs which does not always render them feasible.

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